Contention in Counting Networks

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Abstract

Implementing *counting networks* [1] on shared-memory multi-processor machines often incurs a performance penalty proportional to the extent to which concurrent processors simultaneously access the same memory location. In this work, we continue the study, initiated in [7], of the dependence of performance, as measured by *contention* [5], on the *width* of the balancers used in such constructions. Our main results are two new constructions of counting networks of widths $p2^k$ and pq^k, for any integers $p, q \geq 2$ and $k \geq 0$, respectively, and corresponding formal contention analyses.

The first construction is an elegant generalization of Batcher's classical bitonic network [3] to widths of $p2^k$, using 2- and p-balancers. This construction significantly improves a previous attempt [2] to constructing counting networks of this width in both size and depth. We provide a sharp contention analysis, based on using recurrence relations and exploiting the recursive structure of the construction. This analysis establishes a tight asymptotic bound with dominant term $nk^2/p2^{k-1}$, in the presence of n concurrent processors, and demonstrates that increasing the width of the bitonic counting network [1] by a constant factor p results to a decrease in contention by the same factor. This implies an interesting trade-off between the amount of hardware and the efficiency of software for implementing bitonic counting networks on multi-processor architectures.

The second construction, uses p-balancers and q-balancers and achieves width of pq^k. It is based on constructing a smoothing network of width pq^k, and cascading it with any sorting network. This construction generalizes one of width $p2^k$ presented in [2]. We establish corresponding upper bounds on contention for this construction.

Both correctness proofs for these constructions are modular and systematic, and consist merely of verifying sufficient conditions for counting networks shown in [4].

We are currently implementing a software simulation of our constructions in a general asynchronous multi-processor machine. We have obtained some initial experimental evidence that, under a variety of circumstances, our constructions outperform in typical contention performance previous ones presented in [1, 6].

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